PROFILE LOSS MODEL FOR LOW-PRESSURE TURBINES

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ABSTRACT

This paper presents a model, based on a semi-empirical approach, to predict profile losses of modern Low-Pressure Turbines.

The model may be used to obtain the absolute value of the profile losses as well as the sensitivities to Reynolds number, Mach number, lift coefficient, etc. Therefore, the Reynolds lapse with altitude can be calculated.

In addition, the model may be used to guide the airfoil design. The lift coefficient and the shape of the loading may be selected to optimise a specific application. As example of the model capability, the paper shows as the highest lift coefficient achievable in aft-loaded Low Pressure Turbine airfoils can be inferred.

In order to complete the paper, the validation of the model against rigs and engine tests of several multistage turbines is shown. As conclusion of that validation, one can state that the model is adequate for prediction of Low-Pressure Turbine efficiencies.

NOMENCLATURE

a	Speed of sound
BL	Boundary Layer
BSD	Back Surface Diffusion (V_{MAX}/V_{TE}-1)
C_{pb}	TE pressure base coefficient
C_V	Velocity coefficient (V/V_{TE})
C_X	Axial Chord
Fr	Reduced Frequency
FSTI	Free Stream Turbulence Intensity
H	BL Shape Factor
LPT	Low Power Turbines
M	Mach Number
p	Pitch Spacing
P	Total Pressure
Rg	Gas constant
R	Stagnation Pressure Loss Parameter
Re	Reynolds number based on suction side length and exit conditions
S	Suction side length
SLTO	Sea Level Take off
t	Trailing edge thickness
U	Velocity
V	Velocity
VR	Velocity Ratio
Y_P	Stagnation Pressure Loss coefficient

Greek

α Flow Angle, measured from the turbine axis.
γ Gas specific heat ratio
δ Unguided flow deviation at TE.
δ^* Wake/BL Displacement Thickness
ΔH Stage Total Enthalpy Drop
$\Delta\theta$ Wake Momentum Thickness increment due to TE thickness
ΔS Entropy Increment across blade.
η_R Rotor Row Efficiency
η_{ROW} Row Efficiency
η_S Stator Row Efficiency
η_{STAGE} Stage Adiabatic Total to Total Efficiency
θ Wake/BL Momentum Thickness
ω Kinetic Energy Loss Coefficient $(1-V_3^2/V_{3is}^2)$

Subscripts
0 Stagnation.
1 Cascade Inlet
2 Cascade Exit
3 Cascade Exit Mix Out State
is Isentropic
corr Corrected
ref Reference
P Cascade Pressure Side
R Rotor
S Stator
S Cascade Suction Side
TE Blade Trailing Edge
Throat Blade Throat (Suction Side Velocity Peak)

INTRODUCTION

In the component model described in this paper, the physical mechanisms that govern the entropy generation in low pressure turbine profiles are reproduced by simplified analytical models, with exception of the development of the boundary layer over the rear part of the suction side that is predicted by a correlation obtained from a large experimental database of turbine airfoils.

The model is applicable from Reynolds numbers of $6x10^4$ to $4x10^5$ and from incompressible Mach numbers to high-subsonic (Mach ~ 0.8). A wide range of adverse pressure gradients, over the rear part of the suction side, have been correlated; from null gradients, corresponding to low lift coefficients, to very high gradients, corresponding to ultra high lift airfoils. The effect of the unsteadiness, due to the incoming wakes of upstream rows, on the transition of the suction side boundary layer has been retained through the reduced frequency.

The need of this type of models or correlations in the design of LPT is still important during the early stages of turbine design. When the airfoil geometry does not exist or it is very preliminary, these models may be used to assess the efficiency that will be achievable later, by a more mature design. In addition, for a proper optimisation of the turbine efficiency, they may be used to obtain the trends with the main parameters.

In the open literature many similar loss models can be found for turbines. Starting from the classic component models by Ainley and Mathieson (1951), Dunham and Came (1970), Craig and Cox (1971), among others, followed by more modern ones as Kacker and Okapuu (1981) or Moustapha et al. (1990). More recently Zhu and Sjolander (2005) presented a reviewed version of the Kacker and Okapuu loss system. This paper does not intend to re-examine any of those existing models.

Therefore, the semi-empirical model described in this paper does not intend to be a complete description of the mechanisms that govern the physics of a LPT, but a simple method for calculating trends and efficiency values. As result, overall performance charts, like the Smith's one (1965), can be obtained from the model (see Vazquez et al., 2003).

Finally, much attention has been paid in the last two decades to increase the lift coefficient of turbine airfoils, Curtis et al. (1996), among many others. Successive increases have been achieved through the so-called high lift, ultra high lift airfoils, and so on. Many works are still claiming even higher levels of lift, however little has been written about what is the optimum lift coefficient, corresponding to the minimum profile loss. The present paper shows that based on the presented model, the optimum lift coefficient is between 1 and 1.1, for the characteristic Reynolds numbers of large LPT. Also, it suggests that it may be challenging to exceed this value even with mechanisms of boundary layer control.

In the author's view, the model is proven enough, since more than 200 experiments, linear and annular cascades, multistage cold rigs and engine test data were used for calibration and validation. The accuracy of the efficiency measurement in those experiments varied between ±0.5% in the real engine tests, ±0.2% in the multistage cold flow rigs and finally ±0.1% in the cascades.

MODEL DESCRIPTION

Stage Efficiency Formulation

The adiabatic total to total efficiency of a stage is defined as:

$$\eta_{STAGE} = \frac{\Delta H}{\Delta H_{is}} \tag{1}$$

For an ideal gas with constant γ in each row:

$$\eta_{STAGE} = \frac{(\gamma_R - 1)\cdot \frac{\Delta H}{a_{01}^2}}{1 - \left(1 - (\gamma_R - 1)\frac{\Delta H}{a_{01}^2}\right)\cdot R_S \cdot R_R} \tag{2}$$

Being R_S and R_R the stagnation pressure loss parameter, expressed as:

$$R_S = 1 - \frac{\gamma_S - 1}{2} M_{2S}^2 \left(\frac{1}{\eta_S} - 1\right) \qquad R_R = 1 - \frac{\gamma_R - 1}{2} M_{2R}^2 \left(\frac{1}{\eta_R} - 1\right) \tag{3}$$

And $\eta_{ROW} = 1 - \omega$

Defining the kinetic energy loss coefficient as:

$$\omega = 1 - \frac{V_3^2}{V_{3is}^2} \tag{4}$$

The kinetic energy loss can be classified in the following components : $\omega = \omega_{2D} + \omega_{3D} + \omega_{OTL} + \omega_{REST}$

- $\boldsymbol{\omega_{2D}}$: 2D loss or profile loss is the kinetic energy loss generated by the approximately two-dimensional flow that is present in the greater portion of the span of typical LPT airfoils, usually of high aspect ratio.

- ω_{3D}: 3D loss or secondary flow loss is the kinetic energy loss generated by the flow close to the end-walls that is strongly three-dimensional.
- ω_{OTL}: Over Tip Leakage loss is the stagnation pressure loss generated by the flow that goes through the tip clearance. As LPT use to be shrouded, this clearance is the gap between the blade fins and the honeycomb of the casing. This flow skips the rotor not producing work and mixes back with the main stream generating entropy.
- ω_{REST}: The rest of losses are grouped under a forth term.

For the LPT airfoils that are characterised by high aspect ratios, approximately two thirds of the kinetic energy losses come from the two-dimensional region (ω_{2D}). Thereby, most of the research effort in the past has been oriented in reducing this source of losses.

Strictly speaking the flow in the mid-span region can not be exactly two-dimensional. Factors like the radial variation of pressure may cause a three dimensional radial migration of flow. However, in the central region of the airfoils, the quasi-2D approach is accurate enough in most of the cases. This simplification consists in considering that the flow is contained in a circumferentially averaged streamline surface. Thus, the flow in each airfoil profile is assumed to be equivalent to a two-dimensional flow over the averaged streamline surface, which is analogous to a linear cascade. This justifies the use of linear cascades for validation and calibration of design concepts.

Airfoil Loss Coefficients

Commonly, the merit parameter used in low speed turbine cascades is the loss coefficient or stagnation pressure loss coefficient, defined as:

$$Y_P = \frac{P_{o1} - P_{o3}}{P_{o1} - p_3} = \frac{\Delta P_{o13}}{P_{o1} - p_3} \quad (5)$$

The flow state 1 refers to the inlet of the cascade and state 3 is the mixed out exit flow, where the exit variables at state 2 (typically 50% chord downstream) isentropically are brought to a uniform state by applying the conservation equations of mass, momentum and energy, see figure 1.

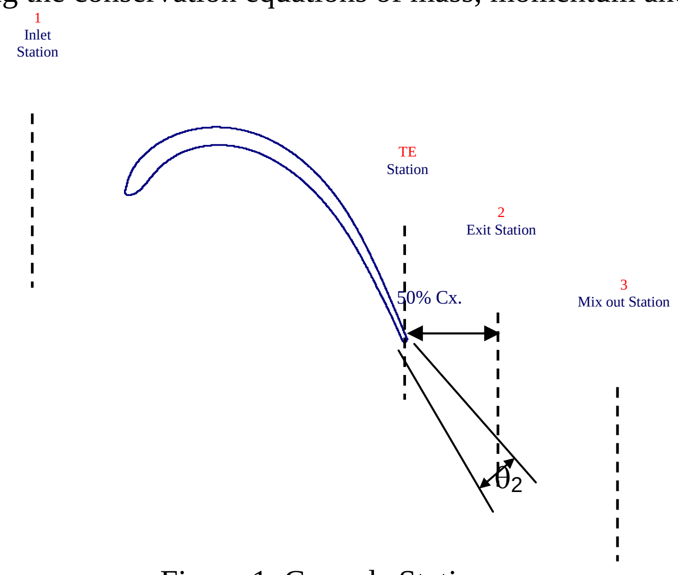

Figure 1. Cascade Stations

The kinetic energy loss (4) is due to an entropy generation through the stage. The entropy generation can be written as function of the stagnation pressure loss coefficient (5), for adiabatic flow and ideal gas in a stationary blade row, according to:

$$\Delta s = -Rg \ln\left(1 - \frac{\Delta P_{o13}}{P_{o1}}\right) = -Rg \ln\left(1 - Y_P\left(1 - \frac{p_3}{P_{o1}}\right)\right) = -Rg \ln\left(1 - Y_P\left(1 - \left(1 + \frac{\gamma - 1}{2} M_{3is}^2\right)^{-\frac{\gamma}{\gamma-1}}\right)\right) \quad (6)$$

Finally, the relation between the kinetic energy loss coefficient (4) and the stagnation pressure loss coefficient (5), in the limit of low losses (as it is usually the case in LPT operating in cruise conditions), can be expressed as:

$$\omega = f(Y_P, M_3) \xrightarrow{Y_P << 1} \omega = \frac{2}{\gamma} \cdot \left(1 - \frac{p_3}{p_{03}}\right) \cdot \frac{1}{M_3^2} \cdot Y_P = \frac{2}{\gamma} \cdot \left(1 - \left(1 + \frac{\gamma - 1}{2} M_3^2\right)^{-\frac{\gamma}{\gamma-1}}\right) \cdot \frac{1}{M_3^2} \cdot Y_P \quad (7)$$

If in addition Mach number is very low, low speed, both coefficients will be the same: $\omega = Y_P$

Profile Loss Formulation

In order to be able to predict the kinetic energy loss coefficient (4), the first step will be express that coefficient as function of the airfoil parameters: solidity, velocity triangles, profile loading, Reynolds No., etc.

Let us consider the mixing of an isolated wake in a channel of height unity, the kinetic energy loss coefficient for low speed can be expressed as, (White, 1991):

$$\omega = \frac{2\theta + \delta^{*2}}{1 - 2(\delta^* - \theta) + 2\delta^{*2}} \quad (8)$$

Assuming a thin wake ($\delta^* << 1$ and $\theta << 1$)

$$\omega = 2 \cdot \theta + (H^2 + 4H - 4) \cdot \theta^2 + O(\theta^3) \quad (9)$$

The order of magnitude of the last term is equal or lower than θ^3, which is much lower than the other two terms. At distances around 50% of the axial chord downstream of the TE, the shape factor of a wake varies from 1 to 1.3 in LPT typical airfoils, then the term θ^2 is negligible. Therefore, it is quite accurate to assume for simplicity: $\omega = 2 \cdot \theta$

Hence, in a turbine airfoil, with a blade height that can be assumed as unity and with a blade spacing of '$p \cdot \cos\alpha_2$', the loss coefficient will be:

$$\omega = \frac{2 \cdot \theta_{WAKE}}{p \cdot \cos\alpha_2} \quad (10)$$

This expression has been derived for incompressible flow and low loss airfoils. At compressible conditions there is no simple equivalent expression that relates the kinetic energy loss coefficient (4) with the airfoils parameters. However, in all the compressible experiments that have been used for this model, the error in the kinetic energy loss coefficient ($\Delta\omega/\omega$) for using (10) was lower than 3% that is considered acceptable. Therefore from now on, the expression (10) will also be used for compressible cases.

Using the length of the suction side (S) as the reference length, equation (10) can be expressed as:

$$\omega = \frac{2 \cdot \frac{\theta_{WAKE}}{S}}{\frac{p}{S} \cdot \cos\alpha_2} \quad (11)$$

The denominator is just a geometric parameter, non-dimensional throat, that is known for a given profile or can be easily estimated in conceptual design from a meanline or a throughflow solver, even when the geometry is still not available. The numerator is a function of the airfoil design style and hence it does not depend on the velocity triangles. It can be considered as the loss of one isolated airfoil of the row; however ω is the losses of the complete ring of airfoils.

The wake momentum thickness at station 2 (see figure 1) depends on the development of the boundary layers over the airfoil surfaces and the wake evolution from the TE. On the other hand, the boundary layer parameters at TE can depend on the velocity or pressure distribution over the airfoil surfaces (Cv), Reynolds number, reduced frequency and the free stream turbulence intensity (FSTI). These four parameters determine the boundary layer growth, the transition to turbulent, separation and reattachment. Based on the accumulate evidences found in all the experiments that have been used for this loss model, one can state that within a representative range for LPT's, the loss coefficient sensitivity to the incoming wakes properties, as amplitude, peak of turbulent intensity, inclination with regard to the turbine shaft, etc., is very weak and therefore one can discard those variables for the model.

If now one assumes that the wake generation and evolution from the TE to the station 2 also depend on: the trailing edge thickness (t/S) and the unguided flow deviation at TE (δ). The following functional dependency can be obtained: $\theta_{WAKE}/S = f_1(\mathrm{Re}\ , Cv, Fr, FSTI, t/S, \delta)$ (12).

The main conclusion from expressions (11) and (12) is that two airfoils with same aerodynamics (same Re, Cv, Fr, FSTI, t/S and δ) and different velocity triangles or/and solidity, have the same wake. This is called the "Cv assumption" and it allows separating the complete row or cascade effects from the isolated airfoil effects.

This principle is very helpful to compare several experiments and also to fix design criteria for a turbine. As it well known, the velocity triangles can vary significantly through the turbine or/and through the airfoil span. On all those cases, the "Cv assumption" can be used to choose the optimum velocity/pressure distribution over the blade surface (Cv), independently of the solidity and of the velocity triangles.

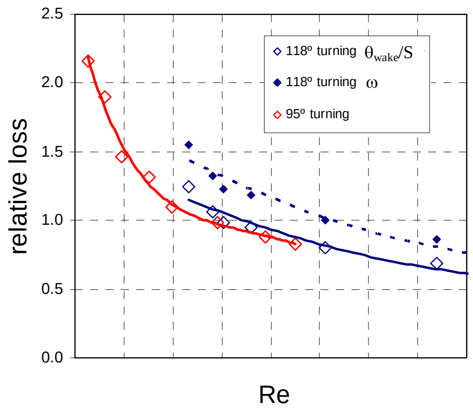

Figure 2. Cv Assumption, Normalised Profile loss for two linear cascades with same Cv and different non-dimensional throat.

The former assumption was validated experimentally. Two linear cascades with identical velocity distribution over the blade surface (Cv) and very different exit flow angle and solidity (different non-dimensional throat) were tested, one of them with a flow turning of 118º (dashed blue line) and the other with a flow turning of 95º (solid red line). Figure 2 shows the results of these cascades. The normalized kinetic energy loss coefficients of those cascades are different. However, when the loss of one of them (dashed blue line) is obtained from expression (11), taking its corresponding θ_{WAKE}/S and the non-dimensional throat of the other cascade (solid red line), the new loss coefficient (solid blue line) agrees well with the loss coefficient of the other cascade (solid red line). This confirms the Cv assumption and indicates that θ_{WAKE} /S is independent of the velocity triangles and therefore it is the right parameter to correlate with.

Wake Generation and Acceleration

The process of wake generation and acceleration is also complex. However the entropy generated in this process is small compared with the one generated in the boundary layers over the airfoil. Typically, it is around a 10% of the overall stagnation pressure losses, based on the experimental data available. For this reason, simple models, even though they may not be very accurate do not introduce a big error in the overall loss estimation.

Even when the wake is accelerated during its generation and mixing process, both effects will be decoupled in the model for simplicity. Firstly, the generation and development of the wake will be considered in a channel with constant area. Afterwards, once the wake is fully developed, shape factor close to unity, it will be accelerated due to the potential static pressure field produced by the adjacent airfoils.

For the first step the equation derived by Denton (1993) is considered:

$$\omega = \frac{2\cdot\theta_s}{p\cdot\cos\alpha_2} + \frac{2\cdot\theta_p}{p\cdot\cos\alpha_2} + \left(\frac{\delta_s^* + \delta_p^* + t}{p\cdot\cos\alpha_2}\right)^2 + \frac{C_{pb}\cdot t}{p\cdot\cos\alpha_2} = \frac{2\cdot\theta_{wake2'}}{p\cdot\cos\alpha_2} \tag{13}$$

This equation comes from conservation of mass and momentum in the mixing of the boundary layers in a blade exit channel with constant area, under certain assumptions: incompressible flow, uniform flow at throat plane, no turning downstream the throat and constant pressure over unguided part of suction side. The station 2' is an idealized flow state located between the TE and the station 2, see figure 1, where the wake is fully developed (H_{wake}=1).

If expression (13) is applied for the reference trailing edge thickness $(t/S)_{ref}$ and for the real value (t/S), subtracting and operating:

$$\left(\frac{\theta_{wake2'}}{S}\right)_{(t/S)} = \left(\frac{\theta_{wake2'}}{S}\right)_{(t/S)_{ref}} + \left(\frac{\Delta\theta}{S}\right)_{\Delta t} \quad \text{and} \quad \left(\frac{\Delta\theta}{S}\right)_{\Delta t} = \left(\overline{H}\cdot\frac{\theta_{BL}}{S} + \frac{1}{2}\cdot\left(\left(\frac{t}{S}\right) + \left(\frac{t}{S}\right)_{ref}\right) + \frac{C_{pb}}{2}\cdot\frac{p}{S}\cdot\cos\alpha_2\right)\cdot\frac{\left(\frac{t}{S}\right) - \left(\frac{t}{S}\right)_{ref}}{p/S\cdot\cos\alpha_2} \tag{14}$$

This term is the trailing edge thickness correction. It computes the impact for having a trailing edge thickness different than the average value of the experimental data base. Other loss models try to calculate the absolute value of the trailing edge loss. The advantage of the present model is that the main part of the trailing edge loss $(\theta_{wake2'}/S)_{(t/S)ref}$ is included in the empirical correlation, so the error for using a simple analytical model is smaller, as long as the thickness of trailing edge is close to the reference value.

The trailing edge correction has three components: contribution of profile boundary layers to blockage, trailing edge thickness blockage and base pressure term. Measurements over a high lift cascade by Hodson and Schulte (1996) show that the split between the three terms is approximately: 25%, 50% and 25%. Thus, the trailing edge blockage is the biggest term and it can be exactly calculated. For the remaining terms some simple assumptions can be made: C_{pb} = 0.04 and H = 2, which corresponds with representative values obtained from the experimental set of data.

For the second step, acceleration of the wake, let us integrate the Karman integral momentum equation for the wake:

$$\frac{d\theta}{\theta} = -\left(2 + H - M^2\right) \cdot \frac{dU}{U} \qquad (15)$$

Assuming unity shape factor and constant Mach number:

$$\frac{\theta_{wake\ 2}}{S} = \frac{\theta_{wake\ 2'}}{S} \cdot VR_{wake}^{\ 3-M^2} \qquad (16)$$

where $VR_{wake}=V_{2'}/V_2$ is the flow velocity ratio between the state 2' and the state 2. Downstream of the TE, in the unguided region, the pressure flow field is highly non-uniform due to the static pressure potential field of the adjacent airfoils. Because of this potential field, the wake suffers an overall acceleration that reduces the thickness of the wake according to equation (16). The negative counterpart is the suppression of the TE velocity that increases the adverse pressure gradient that the suction side boundary layer is subject to.

The value of VR_{wake} can be estimated from CFD. It comes from an inviscid process that can be accurately simulated. In conceptual design, if a representative CFD solution is not still available, a correlation is required to estimate this parameter as a function of: Lift coefficient (through the adverse pressure gradient over the rear part of the suction side), exit flow angle and Mach number.

Description of the Correlation

Once it has been established, as the θ_{wake2}/S is obtained from $\theta_{wake2'}/S$ according with expression (16) and the TE effect corrected by equation (14), the next step will be to obtain $\theta_{wake2'}/S$. In view of expression (12), $\theta_{wake2'}/S$ will only be a function of: $\theta_{WAKE\ 2'}/S = f(\mathrm{Re}, Cv, Fr, FSTI)$ (17), since the effect of unguided deviation and TE have already been quoted.

The parameter C_v represents the isentropic velocity distribution over the airfoil. For similar location of the suction peak velocity and similar pressure side design between airfoils, the velocity distribution can be parameterized only by the level of front loading and by the BSD. The impact of the front loading will be explained in the next section. Let us assume, for the present, that the level of front loading is also similar and then the velocity distribution is only function of BSD, as shown in figure 3. In such case C_v can be replaced by BSD in expression (17), obtaining (18):

$$\theta_{WAKE\ 2'}/S = f(\mathrm{Re}, BSD, Fr, FSTI) \qquad (18)$$

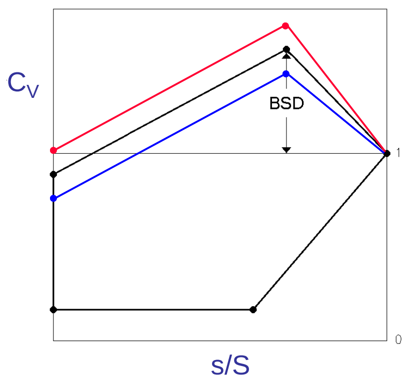

Figure 3. Parametric Velocity Distribution

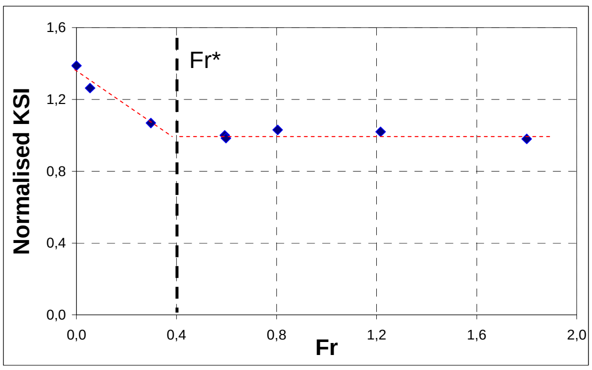

Figure 4. Experimental ω vs. Reduced Frequency at Re $6\text{x}10^4$.

Instead of correlating experimentally every term of equation (13), the presented model uses directly an experimental correlation for $\theta_{wake2'}/S$. A large number of experiments are required to develop the complete correlation. For each experiment, the kinetic energy loss coefficient and the exit flow angle were measured at several Reynolds no. and the value of θ_{wake2}/S was obtained from expression (11) and afterwards it was converted into $\theta_{wake2'}/S$ by extracting the effect of the trailing edge thickness and the acceleration parameter as described above.

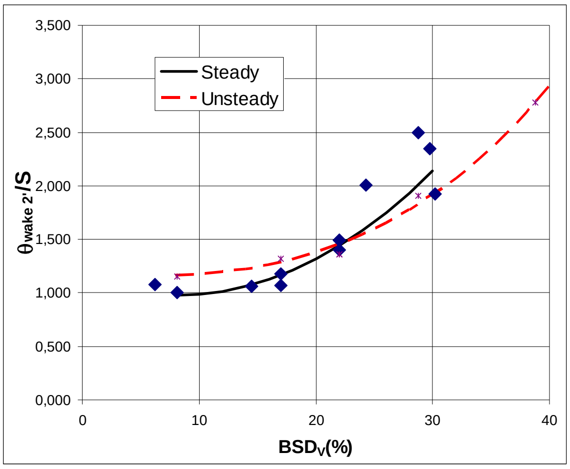

Figure 5, Normalised BL momentum thickness vs. BSD at $Re_S=2.5x10^5$

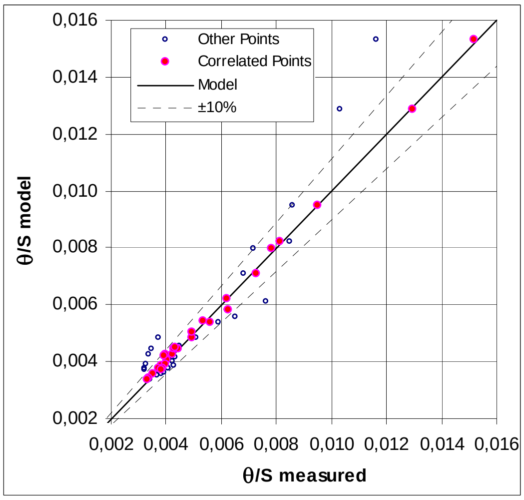

Figure 6. Fitting Error

The FSTI was not correlated, since most of the experiments, where the FSTI was well measured, showed similar levels and maybe lower that those existing in real turbines. On the contrary, the experiments more representative of real turbines, as multistage rigs, did not have a reliable measurement of the turbulence intensity. Therefore, the effect of FSTI on the efficiency and profile losses will be one of the sources of scatter in the correlation.

For the impact of the reduced frequency, two regions have been distinguished in the frequency domain. The first region goes from zero to a critical value (Fr*) and the second region is frequencies above the critical value. The first region that includes the steady solution (Fr = 0) is the limit of low reduced frequencies. In this region, the consecutive wake passage events do not interact in the response of the suction side boundary layer, see Lázaro et al. (2007). Therefore the impact of the reduced frequency on the kinetic energy loss coefficient is lineal.

Most of the LPT airfoils operating under unsteady conditions have frequencies corresponding to the second region. In all those unsteady experiments that have been used for this model, the impact of the reduced frequency within that region was negligible, see figure 4. The variation of kinetic energy loss coefficient ($\Delta\omega/\omega$) was lower than 5% that was considerable acceptable to assume that the loss coefficient is independent of the reduced frequency in that region.

As conclusion, two main set of correlations can be compared, one for unsteady airfoils (Fr > Fr*) and another for steady. The figure 5 shows both correlations as function of the BSD for a specific Reynolds no. In this figure, the dependency of $\theta_{wake2'}/S$ with BSD is clearly seen. These curves for constant Reynolds numbers are fitted with a third order polynomial which coefficients will be different for each Reynolds number.

$$\theta_{wake\ 2'}/S = a(\mathrm{Re}) + b(\mathrm{Re}).(BSD - BSD_{ref}) + c(\mathrm{Re})\cdot(BSD - BSD_{ref})^2 + d(\mathrm{Re})\cdot(BSD - BSD_{ref})^3 \quad (19)$$

It is interesting to remark that the lines of steady and unsteady losses in figure 5 intersect at a certain value of BSD. The physical reason is that unsteadiness reduces the size of the suction side bubbles and this will imply a loss reduction if they are enough large. This phenomenon has been reported by many authors Hodson and Schulte (1996), Lázaro et al. (2007) among others. The BSD value for which both curves intersect will be greater the larger the Reynolds number.

Finally, the fitting error is shown in figure 6, where all measured points of all unsteady airfoils are included. Most of the data fall in the ±10% error band. Therefore, the error of the model is estimated to be a 10% in the absolute value of kinetic energy loss coefficient and it is mostly associated with the uncertainty of the measurements and of course with the assumptions of the model.

The range of application of the present model covers Reynolds numbers from 0.6×10^5 to 3×10^5 and a BSD from 0% to 40% for the unsteady correlation. The upper limit of Reynolds number is extended to 4×10^5 for the steady case.

Front Loading Correction

When the front loading of the profile varies with regard to the one used in the former correlation, a simple correction can be done. Since the flow in the front part is laminar, the momentum thickness increment up to the throat can be computed using Twaites expression (White, 1991):

$$\theta_{throat} / S = 0.67 \cdot \mathrm{Re}_S^{-0.5} \cdot \left[\int_0^{S_{throat}} \left(\frac{U}{U_{TE}} \right)^5 ds \right]^{0.5} \qquad (20)$$

Thus, comparing with the reference velocity distribution:

$$\frac{(\theta_{throat} / S)_{CORR}}{(\theta_{throat} / S)_{REF}} = \left[\frac{\int_0^{S_{throat}} \left(\frac{U}{U_{TE}} \right)^5 ds}{\int_0^{S_{throat}} \left(\frac{U_{REF}}{U_{TE}} \right)^5 ds} \right]^{0.5} \qquad (21)$$

In order to close the problem, the following assumption is considered:

$$\frac{(\theta_{wake\ 2'} / S)_{CORR}}{(\theta_{wake\ 2'} / S)_{REF}} = \frac{(\theta_{throat} / S)_{CORR}}{(\theta_{throat} / S)_{REF}} \qquad (22)$$

This assumption has been validated with the experimental data used for the model. However, that expression is satisfied exactly for the momentum thickness of a boundary layer over the suction side at the TE, if the contribution of friction term was negligible in the Karman integral momentum equation.

Validation of the Model

In this section, results are presented that show the accuracy of the model to predict the efficiency of LPT operating at different conditions. In Figure 7, the calculated turbine efficiency from the model is plotted against the measured efficiency obtained in engine tests, for different LPT. All of them are modern LPT of aero-engines of different thrust range, operating at design conditions of 35Kft of altitude. As it can be seen in this figure, the accuracy of the model is better than 0.5% in overall efficiency, which is considered good enough and agrees with the 10% of error in profile losses shown in the figure 6.

In this validation, the profile loss model described in this paper has been completed with new components for additional losses, as endwall losses, losses related to the over tip leakages, etc. The impact of these new components on the overall turbine efficiency is considered small because they are less than one third of the overall airfoil losses, as it was stated before. The description of those components is out of the scope of this paper.

The model can also be used, as it is shown in figure 8, to calculate the impact of the altitude on the efficiency of LPT, also referred as Reynolds lapse. This figure reproduces the calculated Reynolds lapse from SLTO to high altitude cruise for a modern LPT against the measured in engine. According to these results, the accuracy of the model for this purpose is better than 0.1% in overall efficiency, which is very remarkable, moreover considering that this precision also includes experimental uncertainties.

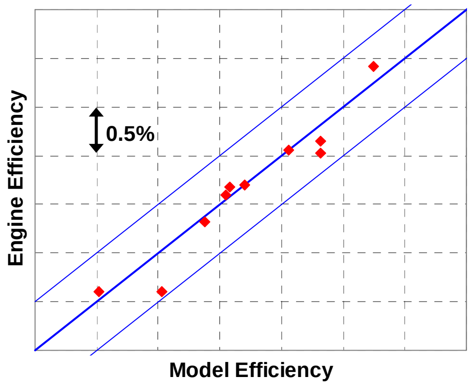

Figure 7. Overall Turbine efficiencies predicted vs. experimental.

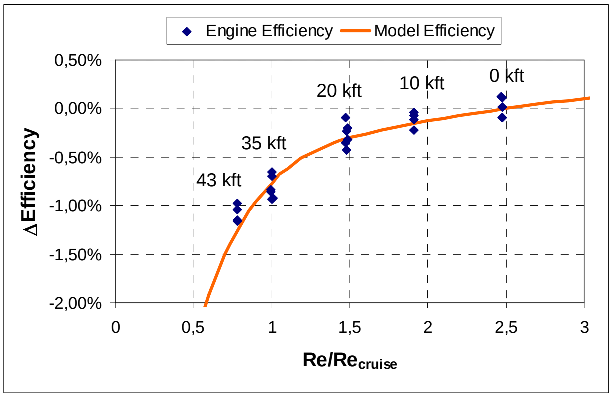

Figure 8. Reynolds number lapse, diamonds: experimental and solid line: calculated

Application of the Model

In the design of a LPT, the model also can be used to select the number of airfoils that produce a minimum value of losses for a given velocity triangles. In this case the pitch-to-suction-side-length ratio (p/S) in equation (11) is not known, as it varies with the lift coefficient. Considering the BSD as the independent variable, the p/S has to be expressed as a function of the BSD. Assuming a parameterization of the pressure distribution as in figure 3 and given a value of BSD, the lift coefficient can be obtained by integration of the pressure curve. From the lift coefficient value, the pitch-to-axial-chord ratio can be calculated. Finally, a geometric correlation that relates the suction side length with the axial chord is required. If the blockage due to the boundary layer, which depends on the level of loss, modified the velocity at TE, the problem would not have an explicit solution and then it should be solved by an iterative procedure. The figure 9 shows an example of lift optimization. The optimum lift coefficient is 0.1 higher if the unsteadiness of the upstream row is retained. At this specific Reynolds number, the unsteadiness implies a small increment of profile loss, as can be seen in figure 9.

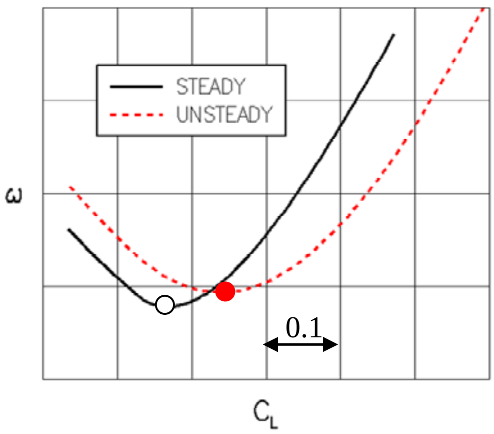

Figure 9. ω vs. Lift Coefficient.

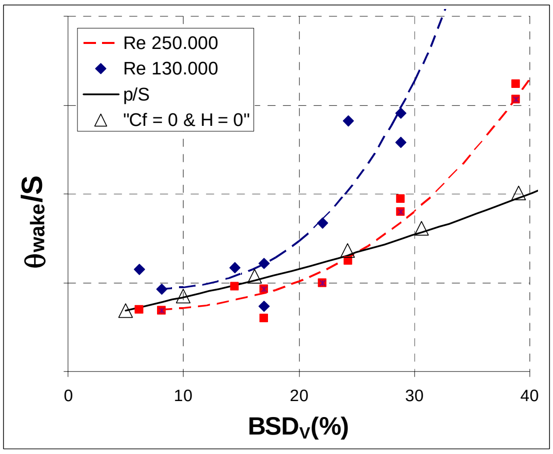

Figure 10. BL momentum thickness vs. BSD

The increase of lift coefficient can be achieved through either a higher BSD (see figure 3) or a higher front loading. In both cases, for a given exit flow angle the denominator of expression (11) increases as the non dimensional pitch (p/S) increases. On the other hand, the numerator will also increase if the lift increases according to figure 5. Therefore the trend of the kinetic energy loss coefficient, ω in expression (11), will depend on which of the two terms grows faster. In the figure 10, the variation of those two terms with the BSD or lift coefficient has been represented. The blue diamonds represents the variation of the numerator (θ_{wake}/S) with the BSD for Reynolds number 1.3x10^5, and the red squares for Reynolds number 2.5x10^5. The solid black line represents the denominator, non dimensional pitch (p/S). The slope of the denominator is lower than the slope of the numerator for BSD's higher than 18-20%. Therefore from this value of BSD, which corresponds to lift coefficients around 1 or 1.1, the losses will increase.

One can think that maybe some mechanism to control the boundary layer could allow a reduction of the slope of the numerator. However the black hollow triangles, which almost fall on the p/S solid black line, represent the variation with BSD of an idealized boundary layer with zero friction coefficient and zero shape factor. Only in this unrealistic case, both the numerator and the denominator grows with the same slope, hence the losses would remain constant as the lift coefficient is increased.

CONCLUSIONS

In this paper, a new model has been presented to predict the profile losses of modern LPT that are the greatest contributor of the loss of performance.

The model has been developed starting from the result of a dimensional analysis and following a semi-empirical approach, where more than 200 experiments, as low and high speed linear cascades, high speed annular cascades, multistage rigs, etc. have been used to support the model.

The applicability range is quite large, being able to be used to predict operating conditions from SLTO to high altitude cruise, incompressible and high subsonic profiles and low lift and ultra high lift airfoils.

The accuracy of the model has been obtained by means of a calibration against data of cascades and whole turbines tested in engines and cold flow rigs, showing a precision better than ±0.25% in the calculation of the absolute efficiency and better than ±0.1% in the estimation of Reynolds lapse.

Some additional applications of the model, as to select the optimum lift coefficient, have been shown. The value of the optimum lift within the range of Reynolds No. studied, under unsteady conditions and for a given representative front loading, turned out to be 1.1.

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